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ID _____

Midterm 3 - 60 Points

You must answer all questions. Please write your name on every page. The exam is closed book and closed notes. You may use calculators, but they must not be graphing calculators. Do not use your own scratch paper.

You must show your work to receive full credit

Problem 1 (45 Points)

Suppose that you wish to predict wage outcomes via the following specification:

$$wage = \beta_0 + \beta_{educ}educ + \beta_{exper}exper + \beta_{IQ}IQ + \beta_{Age}Age + u$$

wage is measured in dollars per month, educ and exper are measured in years. The results from estimating this equation (using the urban subsample of WageData.TXT) are the following:

Call:

```
lm(formula = wage ~ educ + exper + iq + age, data = subset(x,
  urban == 1))
```

Coefficients:

	Estimate	Std. Error	t value	Pr(> t)
(Intercept)	-944.044	195.966	xxxxxxxxxxxxxxxxxxxx	xxxxxxxxxxxxxxxxxxxx
educ	58.218	8.524	xxxxxxxxxxxxxxxxxxxx	xxxxxxxxxxxxxxxxxxxx
exper	11.300	4.473	xxxxxxxxxxxxxxxxxxxx	xxxxxxxxxxxxxxxxxxxx
iq	5.250	1.118	xxxxxxxxxxxxxxxxxxxx	xxxxxxxxxxxxxxxxxxxx
age	15.107	5.695	xxxxxxxxxxxxxxxxxxxx	xxxxxxxxxxxxxxxxxxxx

 Residual standard error: 374.3 on 666 degrees of freedom
 Multiple R-squared: 0.1832, Adjusted R-squared: 0.1783
 F-statistic: 37.35 on 4 and 666 DF, p-value: < 2.2e-16

a.) Please interpret the intercept. Is the value of the intercept meaningful? Why or why not (at least two reasons)?
(5 Points)

On average, a person with 0 education, 0 experience, 0 iq, and of age 0 will earn

\$944 per month ⁺³

No, not meaningful

1. negative	+ 2
2. Age = 0	
3. IQ = 0	

b.) Using the 95% confidence level, test whether the coefficient on $educ$, β_{educ} , is significantly different from zero. Please state your null and alternative hypotheses, and briefly interpret the result. (10 Points)

$$H_0: \beta_{educ} = 0 \quad t_{crit} = 1.96 \quad +2$$

$$+2 \quad H_1: \beta_{educ} \neq 0 \quad t_{stat} = \frac{58.2 - 0}{8.52} = 6.83 \quad +2$$

$$|t_{stat}| > t_{crit} \Rightarrow \text{Reject Null} \quad +2$$

Education has a positive and statistically significant effect on wages $+2$

c.) Please construct a 92% confidence interval for β_{IQ} . Please interpret this confidence interval. (10 Points)

$$t_{crit} = 1.75 \quad +2$$

$$\hat{\beta}_{IQ} - se(\hat{\beta}_{IQ}) \cdot t_{crit} < \beta_{IQ} < \hat{\beta}_{IQ} + se(\hat{\beta}_{IQ}) \cdot t_{crit} \quad +2$$

$$5.25 - 1.12 \cdot 1.75 < \beta_{IQ} < 5.25 + 1.12 \cdot 1.75 \quad +4$$

$$3.325 < \beta_{IQ} < 7.175$$

IQ has a positive and statistically significant effect on wages. $+2$

d.) Suppose I claim that age has no effect on wages. What is the probability that I'm wrong? Please state the null and alternative hypotheses, and show your work! (10 points)

$$+2 \quad P_{value} = Pr(|T| > t_{stat})$$

$$H_0: \beta_{age} = 0 \quad +2$$

$$+2 \quad = 2 \cdot (1 - Pr(T < t_{stat}))$$

$$H_1: \beta_{age} \neq 0$$

$$= 2 \cdot (1 - 0.996)$$

$$t_{stat} = \frac{15.107}{5.695} = 2.65$$

$$= 2 \cdot (0.004) = 0.008$$

+4

.008 probability of being wrong

e.) Please derive a new estimating equation that will generate a prediction and standard error for a 50 year old person with 15 years of education, 20 years of experience, and a 140 IQ. Show your work!!! (10 Points)

$$\theta = \beta_0 + \beta_{\text{educ}} \cdot 15 + \beta_{\text{exper}} \cdot 20 + \beta_{\text{IQ}} \cdot 140 + \beta_{\text{Age}} \cdot 50 + 2$$

$$\Rightarrow \beta_0 = \theta - \beta_{\text{educ}} \cdot 15 - \beta_{\text{exper}} \cdot 20 - \beta_{\text{IQ}} \cdot 140 - \beta_{\text{Age}} \cdot 50 + 2$$

$$\text{wage} = \beta_0 + \beta_{\text{educ}} \cdot \text{Educ} + \beta_{\text{exper}} \cdot \text{Exper} + \beta_{\text{IQ}} \cdot \text{IQ} + \beta_{\text{Age}} \cdot \text{Age}$$

$$= \theta - \beta_{\text{educ}} \cdot 15 - \beta_{\text{exper}} \cdot 20 - \beta_{\text{IQ}} \cdot 140 - \beta_{\text{Age}} \cdot 50 + 2$$

$$+ \beta_{\text{educ}} \cdot \text{Educ} + \beta_{\text{exper}} \cdot \text{Exper} + \beta_{\text{IQ}} \cdot \text{IQ} + \beta_{\text{Age}} \cdot \text{Age}$$

$$\text{wage} = \theta + \beta_{\text{educ}} (\text{Educ} - 15) + \beta_{\text{exper}} (\text{Exper} - 20)$$

$$+ \beta_{\text{IQ}} (\text{IQ} - 140) + \beta_{\text{Age}} (\text{Age} - 50)$$

+11

Problem 2 (15 Points)

In economics, it is common to assume that production is "Cobb-Douglas". Assuming capital and labor are the only inputs, the Cobb-Douglas production function is written as follows:

$$Y = \exp(\beta_0) \cdot \exp(\varepsilon) \cdot K^{\beta_K} \cdot L^{\beta_L}$$

Here, Y represents production, β_0 is a constant, ε is a random productivity shock, K is capital used in production, L is labor used in production, and β_K and β_L are parameters related to capital and labor.

Economists often wish to estimate production, and in particular, the parameters β_K and β_L . To do so, we take logs to get:

$$\log(Y) = \beta_0 + \beta_K \log(K) + \beta_L \log(L) + \varepsilon$$

This production function exhibits "constant returns to scale" if $\beta_K + \beta_L = 1$. Please derive an estimating equation that allows you to test whether production exhibits constant returns to scale. Be sure to write down your null and alternative hypotheses. Show your work!!!

Define $\theta = \beta_K + \beta_L$ + 2

~~$H_0: \theta \neq 1$~~
 ~~$H_A: \theta = 1$~~

$H_0: \theta = 1$ + 2

$H_A: \theta \neq 1$

$$\beta_L = \theta - \beta_K$$

$$\log(Y) = \beta_0 + \beta_K \log(K) + (\theta - \beta_K) \log(L) + \varepsilon$$
 + 2

$$\left(\log(Y) = \beta_0 + \beta_K (\log(K) - \log(L)) + \theta \log(L) + \varepsilon \right)$$
 + 4